## Exercise 31

Evaluate the integral.

$$\int_{\pi/6}^{\pi/2} \csc t \cot t \, dt$$

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**Solution**

Use the formula for the derivative of cosecant on page 193 to rewrite the integrand. Then use part 2 of the fundamental theorem of calculus to evaluate the integral.

$$\begin{aligned}
\int_{\pi/6}^{\pi/2} \csc t \cot t \, dt &= \int_{\pi/6}^{\pi/2} \left[ -\frac{d}{dt}(\csc t) \right] dt \\
&= -\int_{\pi/6}^{\pi/2} \frac{d}{dt}(\csc t) \, dt \\
&= -(\csc t)\Big|_{\pi/6}^{\pi/2} \\
&= -\left( \csc \frac{\pi}{2} - \csc \frac{\pi}{6} \right) \\
&= -\left( \frac{1}{\sin \frac{\pi}{2}} - \frac{1}{\sin \frac{\pi}{6}} \right) \\
&= -(1 - 2) \\
&= 1
\end{aligned}$$